Power Options
in Executive Compensation

Carole Bernard[†], Phelim Boyle[§] and Jit Seng Chen[‡] [*]

[†]*Department of Accounting, Law and Finance, Grenoble Ecole de Management.*

[§]*School of Business and Economics, Wilfrid Laurier University.*

[†] *GGY AXIS, Toronto.*

Email: carole.bernard@grenoble-em.com, pboyle@wlu.ca, jitseng.chen@gmail.com

February 5, 2016

Abstract

Tian [2013] shows that firms are better off linking incentive pay to average stock prices. This paper proposes a new type of executive stock option contract that improves upon Tian's Asian executive option. This contract is a type of power option and its price has a simple closed-form expression under standard Black-Scholes assumptions. Numerical results show that the new contract is on average four percent cheaper than Tian's. Holding constant the company's cost of option issuance, the power option is more valuable to executives and has superior incentive properties compared to Tian's Asian executive option and to the standard call option.

Key-words: Executive compensation, Cost-efficiency, Asian options, Optimal contracting, Incentive effects.

JEL classifications: G13, G30, J33, M52.

1 Introduction

Stock options are an important component of executive compensation in many companies. This paper shows how to design a new type of executive stock option that has advantages over the Asian options introduced by Tian [2013]. Our paper contributes to the existing literature on the design of executive options and is similar in spirit to Johnson and Tian [2000b] who analyze a range of different executive option designs.

Tian [2013] has shown that Asian options have advantages over traditional call options as a form of executive compensation. Using a principal-agent analysis, he concludes that firms are better off basing executive options on average stock prices than basing them on stock prices at maturity. In general, the use of averaging is found to be more cost-effective and incentive-effective than the use of traditional stock options. This means that for a given cost of the option grant, risk-averse executives benefit from a higher subjective value of the Asian option than the European one. On the other hand, incentive-effective means that Asian-style options provide executives with stronger incentives to increase the stock price than standard ones.

Our main contribution is to design a new type of executive option (a power option) that dominates an Asian executive option in several respects. While an Asian option payoff is based on the average path of the stock price, the power option payoff is essentially based on a power of the terminal stock price. The power option has the same ex-ante distribution as the Asian option but it it is cheaper. It is constructed using the concept of cost-efficiency, which was introduced by Dybvig [1988a] and then extended by Bernard et al. [2014].

We compare the properties of the proposed power options with Asian options and standard call options. We show that the power option is on average 4% cheaper than Tian's [2013] Asian option. We also show that for the same option cost, the subjective value of the power option is strictly higher than that of the Asian option. We demonstrate that the power option dominates the Asian option in the case of two out of three incentive measures. The power option is superior in terms of incentives related to manipulating the stock price and volatility but not in terms of reducing dividends. We argue that this is

not an issue given the reluctance of companies to reduce dividends even under adverse conditions.

The rest of this paper is organized as follows. Section 2 sets out our framework and reviews Tian's [2013] design. This section explains cost-efficiency through an example. Section 3 describes our new design. Section 4 compares the properties of our proposed contract with Tian's Asian option and with a standard call option. In Section 5, we discuss a number of important extensions and limitations of the analysis. Appendix A contains some expressions that are required to compute pay for performance sensitivities. All proofs are presented in Appendix B. To make the paper more readable, some of the longer tables are placed in Appendix C.

2 Framework

This section lays out our framework. We explain certain concepts that are needed in the paper. Specifically, we start with the valuation of executive options, and then recall the concept of cost-efficiency.

In the context of executive options, we can distinguish between the cost to the company and the value to the executive. The cost to the company is the risk-neutral expectation of the discounted payoff of the executive option. The value to the executive is referred to as the subjective value of the contract (Lambert et al. [1991]). Similar to Tian [2013], we assume that the executive's total wealth comprises of a fixed salary and stock options, and use the certainty equivalent of the total wealth to derive the subjective value of the option. Let W_0 be the initial wealth and λ be the fraction of the executive's initial wealth in the form of stock options. Let n be the number of options issued in the compensation package such that $n = \frac{\lambda W_0}{c(V_T)}$ where $c(V_T)$ is the initial cost of one option with payoff V_T to the company. The final wealth W_T at maturity T is given by

$$W_T = (1 - \lambda)W_0 e^{rT} + nV_T$$

where V_T denotes the payoff of a single option.

The certainty equivalent (CE) is defined as the amount of cash that an executive with an increasing utility function $u(\cdot)$ would view as equivalent in value to the stock options. In other words, the executive is indifferent (same expected utility level) between the amount of cash CE and the random payoff associated with the compensation in options, nV_T. Specifically, CE is defined as

$$u\left(((1-\lambda)W_0 + n\text{CE})e^{rT}\right) = E_{\mathbb{P}}\left(u(W_T)\right)$$

where the expectation is under the agent's subjective probability measure $\mathbb{P}$. Without loss of generality,[1] we get

$$u\left(1 - \lambda + \lambda\frac{\text{CE}}{c(V_T)}\right) = E_{\mathbb{P}}\left[u\left(1 - \lambda + \lambda\frac{V_T}{c(V_T)}e^{-rT}\right)\right] \tag{1}$$

The subjective value of a particular option is defined as the fraction of the certainty equivalent CE of the option with respect to its risk-neutral value $c(V_T)$ i.e. $\frac{\text{CE}}{c(V_T)}$.

To obtain a cheaper form of option compensation, we use the concept of "cost-efficiency". This concept was introduced by Dybvig [1988a, 1988b] and Cox and Leland [2000],[2] and recently extended by Bernard et al. [2014]. Cost-efficiency refers to achieving a given probability distribution at the lowest possible cost. We use a two-period numerical example to illustrate the basic idea. A more formal treatment can be found in the literature cited above.

## 2.1   Numerical example to illustrate cost-efficiency

Consider the following two-period discrete time model in the Cox, Ross and Rubinstein [1979] framework where the initial stock price is 16 and the real-world probabiliy of an up

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[1] But with a slight abuse of notation in that we replace $x \mapsto u(W_0 e^{rT}x)$ with $x \mapsto u(x)$

[2] The original Cox and Leland paper was written in 1982 but not published until 2000 in Cox and Leland [2000].

jump at each node is $p = \frac{1}{2}$:
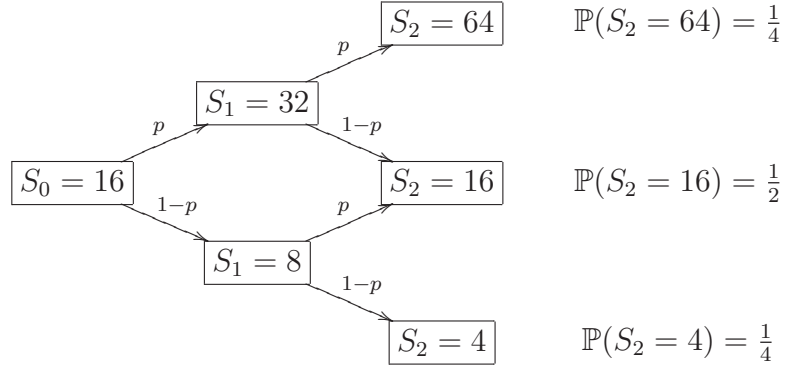
Let V_2 be the payoff at the end of the second period:

$$V_2 = \begin{cases} 1 & \text{if } S_2 = 64, \\ 2 & \text{if } S_2 = 16, \\ 3 & \text{if } S_2 = 4. \end{cases} \tag{2}$$

The expected utility of this payoff can be calculated as $E_{\mathbb{P}}[u(V_2)] = \frac{u(1)}{4} + \frac{u(3)}{4} + \frac{u(2)}{2}$.

When the interest rate is ten per cent per period, the corresponding risk-neutral probability of an up jump is $q = \frac{2}{5}$. Thus the risk-neutral probabilities that $S_2 = 64$, $S_2 = 16$ or $S_2 = 4$ are respectively equal to $\frac{4}{25}$, $\frac{12}{25}$ and $\frac{9}{25}$. We denote by $c(V_T)$ the cost of an executive option with terminal payoff V_T computed as the risk-neutral expectation of the discounted payoff. The time zero cost of the payoff V_2 is given by

$$c(V_2) = \frac{\mathbb{Q}(S_2 = 64)}{(1+r)^2} \times 1 + \frac{\mathbb{Q}(S_2 = 16)}{(1+r)^2} \times 2 + \frac{\mathbb{Q}(S_2 = 4)}{(1+r)^2} \times 3 \approx 1.82.$$

Consider another payoff given by

$$V_2^{\star} = \begin{cases} 3 & \text{if } S_2 = 64, \\ 2 & \text{if } S_2 = 16, \\ 1 & \text{if } S_2 = 4. \end{cases}$$

The key observation is that the expected utility of the second payoff, $E_{\mathbb{P}}[u(V_2^\star)]$ is equal to the expected utility of V_2 whereas its cost, $c(V_2^\star)$, is now strictly lower:

$$c(V_2^\star) \quad = \quad \frac{\mathbb{Q}(S_2 = 64)}{(1+r)^2} \times 3 + \frac{\mathbb{Q}(S_2 = 16)}{(1+r)^2} \times 2 + \frac{\mathbb{Q}(S_2 = 4)}{(1+r)^2} \times 1 \approx 1.49 < c(V_2).$$

In fact, the payoff $V_2^\star$ is cost-efficient, i.e. it is the cheapest way to get the same distribution as the payoff of V_2 under the physical measure because its outcomes are in reverse order to the price of each state.[3] Observe that $V_2^\star$ is non-decreasing with respect to S_2. In the Black-Scholes model, Bernard et al. [2014] show that when $\mu > r$, a payoff is cost-efficient if and only if it is non-decreasing with respect to S_T. This characterization leads to an explicit representation of cost-efficient payoffs that will be useful to construct a better executive option than the Asian executive option proposed by Tian [2013].

2.2 Asian Executive Option

We now briefly describe the payoff of the Asian executive option introduced by Tian [2013].[4] This option has a payoff

$$A_T = (S_T^{\text{avg}} - K)^+ \tag{3}$$

where S_T^{avg} denotes the geometric average of the stock price from time 0 to time T and K is the strike. If the stock price is observed at discrete dates $t_1, t_2,..., t_n$, then the geometric average can be computed as

$$(S_{t_1} S_{t_2} ... S_{t_n})^{\frac{1}{n}}. \tag{4}$$

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[3]It follows from Theorem 1 of Dybvig [1988a], which states that in a complete discrete-time market model where all states are equally probable, *"the cheapest way to achieve a lottery assigns outcomes of the lottery to the states in reverse order of the state-price density."* The state-price process at time t, $t \in \{0, 1, 2\}$ is the ratio of probabilities under the risk-neutral measure and the physical measure discounted at the risk-free rate. Bernard et al. [2014] extend this result to a general discrete-time or continuous-time market.

[4]Tian [2013] also studied indexing. However, Dittmann et al. [2013] investigated the impact of indexing executive options by calibrating the standard principal-agent model to a large sample of US CEO contracts. They find that the benefits from indexing executive options are small and actually increase the compensation costs by 50% for most firms in their sample. They conclude also that the overall benefit of indexing options is ambiguous, which could explain why indexing is rarely used in practice (e.g. Hall and Murphy [2003], Murphy [1999]). For this reason, we will focus on the Asian executive option without indexing.

For convenience, we will use the limiting case where the stock is observed at all times and the geometric average is defined as

$$S_T^{\text{avg}} = \exp\left\{ \frac{1}{T} \int_0^T \ln S_t dt \right\}. \tag{5}$$

From the discussion above, cost-efficient payoffs must be non-decreasing in the final stock price. Thus the Asian option may not be cost-efficient. In this case, we can construct a new option payoff, say P_T with the same probability distribution as A_T under the real-world probability $\mathbb{P}$ but at a strictly smaller cost. Observe that for any utility function u, A_T and P_T have the same expected utility since they have the same probability distribution (that is, $E_{\mathbb{P}}[u(A_T)] = E_{\mathbb{P}}[u(P_T)]$). The rest of the paper will illustrate how to use cost-efficiency to improve CEO compensation schemes. Interestingly, this improvement will not be only in terms of cost of issuance to the company but will also provide a higher subjective value (for a given cost) and better incentives to the executive. The next section discusses this strict Pareto improvement in more detail.

3   The Power Executive Option

In this section, we construct a power option as an alternative to the Asian executive option proposed by Tian [2013]. We use the same notations as Tian [2013]. In this setting, our new option has an explicit expression. Assume a constant risk-free rate r. The stock price process (under the physical measure $\mathbb{P}$) is given by

$$\frac{dS_t}{S_t} = (\mu - q)dt + \sigma dZ_t \tag{6}$$

where $\mu > r$ and q are respectively the expected rate of return and dividend yield, σ is the volatility and Z_t is a standard Brownian motion. At time T, the stock price can be written as

$$S_T = S_0 \exp\left\{ \left(\mu - q - \frac{\sigma^2}{2} \right) T + \sigma Z_T \right\}. \tag{7}$$

Here, the cost of a payoff V_T paid at time T is given by $c(V_T) = E_{\mathbb{Q}}[e^{-rT}V_T]$. Our main result is presented in the next proposition. We give explicit expressions for the power option that dominates the Asian option.

Proposition 1. *Consider the following power option*

$$P_T = \psi \left(S_T^{\varphi} - \frac{K}{\psi} \right)^+ \tag{8}$$

where $\varphi = \frac{1}{\sqrt{3}}$ and $\psi = S_0^{1-\varphi} \exp \left\{ \left(\frac{1}{2} - \varphi \right) \left(\mu - q - \frac{\sigma^2}{2} \right) T \right\}$. It has several important properties.

- *First, the power option with payoff P_T above has the same distribution as the Asian executive option A_T in (3).*

- *Second, the power option with payoff P_T is cheaper than Tian's option with payoff A_T. Specifically, the costs of the Asian option A_T and the power option P_T at time zero are given by*

$$c(A_T) = BSCall[S_0, K, r, T, \varphi\sigma, q_{avg}]$$
$$c(P_T) = BSCall\left[S_0, K, r, T, \varphi\sigma, q_{avg} + \left(\varphi - \frac{1}{2} \right)(\mu - r) \right] \tag{9}$$

where $BSCall[S_0, K, r, T, \sigma, q]$ is the usual Black-Scholes[5] price of a call option (with initial underlying price S_0, strike K, risk-free rate r, maturity T, volatility σ and dividend yield q), and where $\varphi = \frac{1}{\sqrt{3}}$, $q_{avg} = \frac{1}{2} \left(r + q + \frac{\sigma^2}{6} \right)$ and $\varphi - \frac{1}{2} \approx 0.0774$.

- *For any non-decreasing utility function u, the subjective value of the power option accounting for the cost of issuing the option is always higher than the subjective value*

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[5] $BSCall[S_0, K, r, T, \sigma, q] = S_0 e^{-qT} N(d_1) - Ke^{-rT} N(d_2)$ where $d_1 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r - q + \frac{\sigma^2}{2} \right)T}{\sigma\sqrt{T}}$ and $d_2 = d_1 - \sigma\sqrt{T}$.

of the Asian executive option

$$\frac{CE_{Power}}{c(P_T)} > \frac{CE_{Asian}}{c(A_T)}.$$

Proof. Let us first give some intuition about the proof of Proposition 1. The idea is that the probability distribution of S_T^{avg} is LogNormal (Kemna and Vorst [1990]). We then choose ψ and φ such that the payoff ψS_T^{φ} is also LogNormal with the same moments as S_T^{avg} and thus with the same distribution. Specifically, Kemna and Vorst [1990] showed that the price (geometric) average S_T^{avg} has the same distribution as $\widehat{S}_T$:

$$\widehat{S}_T = S_0 \exp\left\{\frac{1}{2}\left(\mu - q - \frac{\sigma^2}{2}\right)T + \varphi\sigma\widehat{Z}_T\right\} \tag{10}$$

where $\widehat{Z}_t$ is a standard Brownian motion under $\mathbb{P}$. We can compare (10) with the expression of the stock price in (7) recalled here

$$S_T = S_0 \exp\left\{\left(\mu - q - \frac{\sigma^2}{2}\right)T + \sigma Z_T\right\}$$

to see that the distribution of $\widehat{S}_T$ has almost the same form as the distribution of S_T, after adjusting the drift and volatility terms. In fact, the drift and volatility adjustments are precisely ψ and φ so that we have

$$\psi S_T^{\varphi} \sim \widehat{S}_T \sim S_T^{\text{avg}}$$

where $\sim$ denotes equality in distribution. The two terms, ψ and φ, are thus needed to adjust the terminal stock price, S_T, so that it mimics the distribution of the underlying Asian option, S_T^{avg}. This removes the geometric averaging component, but preserves the distribution of the average since the presence of φ reduces the stock price volatility. The first statement of the proposition is proved.

The second statement of Proposition 1 follows from cost-efficiency (Bernard et al. [2014]) that the cost of the power option is strictly lower. Cost-efficiency states that the cheapest

way to achieve a given probability distribution is to be increasing in the final value of the stock price (when $\mu > r$) (more details can be found in Appendix B). We thus need to construct an option that has the same distribution as Tian [2013]'s Asian executive option but that is non-decreasing in the terminal stock price. Note also that the cost of the power option, $c(P_T)$, and the cost of the Asian option, $c(A_T)$, share a similar expression (9) based on the Black-Scholes formula. They can therefore be readily compared. The only difference between the two expressions is in the term that takes the place of the dividend yield that is used in the Black-Scholes formula. Given that $\mu > r$, the power option price is computed with a higher dividend yield than the Asian option. It is thus cheaper because a call option price is decreasing in the dividend yield.

The last statement of Proposition 1 is proved in Appendix B. $\qquad\qquad\square$

At first sight, this proposition may look complicated but the power option payoff is based on the ending stock price only and it has a simple design. By construction, its cost of issuance to the company is lower (second statement of Proposition 1). It could be argued that cost-effectiveness of executive options should not be measured by the cost of issuance but by the subjective value to CEOs. However, it turns out that the power option is not only cheaper for the company to issue, but it also has a larger subjective value to the executives if we assume the same dollar expenditure by the company (third statement of Proposition 1). Tian [2013] shows that the Asian option is generally more cost-effective than traditional executive call options. His conclusion is based on the ratio between the certainty equivalent and the cost of issuing the option. Here, we show that the power option always has a larger subjective value. The numerical analysis in Section 4 will quantify the magnitude of this gain and compare it directly with some of the numerical experiments of Tian [2013].

3.1 Comparison with the Asian option

We now provide some additional discussion of the main features of the power option and also how it compares with the Asian option. First, we discuss a numerical example that

illustrates how it operates. Then, we compare the payoffs of the power option, P_T, and of the Asian option, A_T, under a few stock price paths. Finally, we compare the performance of these two options during the last ten years for a particular stock.

The payoff of the power executive option is given by

$$P_T = (\psi S_T^\varphi - K)^+ . \tag{11}$$

There is a threshold stock price above which the power option is in-the-money and below which it is out-of-the-money. We use that as a reference point in our comparison and we denote it by $\widehat{H}$.

$$\widehat{H} = \frac{K^{\frac{1}{\varphi}}}{\psi^{\frac{1}{\varphi}}}. \tag{12}$$

We now find the values of $\widehat{H}$ for some specimen parameters. For our base case we assume that

$$S_0 = 100, K = 100, \sigma = .35, T = 5, r = .04, \mu = .08, q = 0. \tag{13}$$

For this case we find that $\widehat{H} = 101.26$. It is easy to show that $\widehat{H}$ is increasing in μ and decreasing in σ.

Although the payoff of the power option is designed to have the same distribution as the Asian option, it will not necessarily have the same cash-flows. We illustrate this idea with an actual stock: Legg Mason Inc (NYSE:LM). This is an asset management company based in Baltimore, Maryland.

We issue five year at-the-money and in-the-money options of both types[6] on July 1, 2003 and also on July 1, 2008. For each option, we use five years of prior data to estimate the various parameters using maximum likelihood estimation. S_T^{avg} is estimated using the geometric average of daily closing prices (as in (4)). Exhibit 1 depicts Legg Mason's stock price over these two periods.

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[6]These correspond to the geometric Asian option and our proposed power option.

Legg Mason Closing Price from July 1, 1998 to July 1, 2013

Time series plot of daily Legg Mason closing prices from July 1, 1998 to July 1, 2013. Historical prices are extracted from Yahoo finance.

During the first period (2003-2008), the average is quite high because the price rose and fell. However, during the second period (2008-2013), the opposite happened. This is confirmed by the payoffs of the power option and Asian option that are given in the last two columns of Exhibit 2. The Asian option and the power option have different cash-flows at maturity on an ex-post basis even if they share the same ex-ante probability distributions. Different orderings are possible on an ex-post basis. For instance, it is possible to have a positive payoff for the Asian option and a worthless power option (this is the case of the at-the-money option issued on July 1, 2003). This is essentially driven by the fact that the average price is higher but the terminal stock price is lower than the initial stock price at the date of issuance. The opposite situation in which the Asian option expires worthless and the power option has a positive payoff is possible as can be seen from the payoffs of the two in-the-money options issued on July 1, 2008. Both options can also expire worthless (see the two at-the-money options issued on July 1, 2008).

12

Issue	$\mu - q$	σ	K	S_T	$\widehat{H}$	S_T^{avg}	Payoff A_T	Payoff P_T
July 1, 2003, ATM	0.262	0.444	44.63	41.25	49.78	77.94	33.31	0.00
July 1, 2003, ITM	0.262	0.444	30	41.25	25.02	77.94	47.94	10.04
July 1, 2008, ATM	0.053	0.322	41.25	31.31	41.28	27.54	0.00	0.00
July 1, 2008, ITM	0.053	0.322	30	31.31	23.78	27.54	0.00	5.16

EXHIBIT 2:

The Asian and Power options are issued on July 1, 2003 and July 1, 2008 and at-the-money (ATM) and in-the-money (ITM) respectively. $\mu - q$ and σ are estimated using the MLE of five years of prior data.

We observe from Exhibit 1 that at the end of the period, the stock price is low but the average is relatively high. From the company's perspective, it may not be a good decision to pay large option payoffs given that the stock price is low. In this situation, the power option will be more attractive to the company since it is out of the money when the ending stock price is low irrespective of the previous stock price path.

4   Comparison of Executive Options

This section contains a comparison of three executive options (the power option proposed in this paper, the Asian option proposed by Tian [2013], and a standard European call option). In order to provide a more comprehensive review, we select a wide range of option parameters using the framework introduced by Broadie and Detemple [1996]. We first base our discussion on cost-effectiveness (costs and subjective values) and then on the incentive measures used by Tian [2013]. Specifically, we compare these options on the basis of cost (risk-neutral value and subjective value) and of incentives to increase share price and volatility, and reduce share dividends.

Exhibit 3 displays ranges for each parameter needed for our comparison. We assume that these parameters are uniformly distributed within their respective ranges and are independent of each other. The sample of parameters used is constructed by randomly drawing sets of option parameters from these distributions. We only consider parameters

that produce a cost for the Asian executive option $c(A_T)$ of at least 5, which represents five per cent of the initial wealth W_0 that is set to 100. We use a sample of size 10,000.

Parameter	Symbol	Range
Initial share price	S_0	100
Strike price	K	$[50, 150]$
Volatility	σ	$[0.20, 0.60]$
Interest rate	r	$[0.03, 0.06]$
Expected return	μ	$r + [0.01, 0.05]$
Term	T	$[3, 10]$
Dividend yield	q	$[0, 0.02]$

EXHIBIT 3:
Range of values for each parameter. Each set of parameters is sampled uniformly and independently from the above ranges provided that $c(A_T) > 5$. This process is repeated until a sample of 10,000 valid option prices is attained.

4.1 Comparison of costs and subjective values

Here we compare costs and subjective values. This set of results confirms our theoretical findings: the power option P_T is strictly cheaper than the Asian option but provides a higher subjective value to executives holding option expenditure constant. Let us recall the notation for the payoff of the power option, P_T, the payoff of the Asian option, A_T, and let us denote by C_T the payoff $(S_T - K)^+$ of a traditional call option with the same strike as the Asian option. It is then known that

$$c(P_T) < c(A_T) < c(C_T).$$

The first inequality follows from the fact that by construction, the power option is cost-efficient and thus cheaper (Proposition 1). The second part is a well-known inequality between the price of an Asian call and a traditional call with the same strike.

We sample 10,000 parameters from Exhibit 3 and compute option costs for each of these parameters. They are provided in Exhibit 4.

Option prices	Mean	RMSE
Power option $c(P_T)$	20.0306	9.4294
Asian option $c(A_T)$	20.7628	9.6145
Standard call $c(C_T)$	44.7951	12.5801

EXHIBIT 4:
Summary statistics of sample of 10,000 option costs. Payoffs are sorted in decreasing order of the average score.

The "Mean" and "RMSE" columns are the average and root mean square error respectively. Observe that even though the RMSE is relatively large across the board, it does not imply the conclusions drawn are less valid. It means that the sample used is comprehensive enough to capture a wide range of option values, thus allowing the average scores to be more conclusive.

Based on the same sample of 10,000 parameters, we compute the relative efficiency loss of the Asian executive option over the corresponding power option. It is defined as follows based on a comparison of the cost $c(A_T)$ of the Asian payoff A_T and the cost $c(P_T)$ of the power option payoff P_T:

$$\frac{c(A_T) - c(P_T)}{c(A_T)} = 1 - \frac{\text{BSCall}\left[S_0, K, r, T, \varphi\sigma, q_{\text{avg}} + \left(\varphi - \frac{1}{2}\right)(\mu - r)\right]}{\text{BSCall}\left[S_0, K, r, T, \varphi\sigma, q_{\text{avg}}\right]} \tag{14}$$

This quantity measures the extra amount of money spent on the Asian executive option to achieve essentially the same distribution as its cost-efficient alternative (the power option), relative to the cost of the Asian executive option itself. The numerical results are reported in Exhibit 5.

On average, the relative efficiency loss is 3.90% and ranges from a minimum of 0.57% to a maximum of 14.70%. Replacing the Asian option by a power option reduces the option cost. To be precise, the power option is cheaper than the Asian executive option provided that the expected return μ is greater than the risk-free rate r as can be seen clearly from (14). If both parameters μ and r are equal, then the efficiency loss is zero.

Relative Efficiency Loss	Mean	Min	Max	RMSE
$\frac{c(A_T)-c(P_T)}{c(A_T)}$	3.90%	0.57%	14.70%	2.07%

EXHIBIT 5:
Summary statistics of the Relative Efficiency Loss corresponding to the sample of 10,000 option costs.

We next turn to the executive's subjective value of the option. We assume that the executive maximizes the expected utility of wealth at some date T and that she has a CRRA utility function

$$u(w) = \frac{w^{1-\gamma}}{1-\gamma} \tag{15}$$

where w is the level of wealth and $\gamma \geqslant 0$ is the coefficient of relative risk aversion. The case of $\gamma = 1$ corresponds to log utility.

In addition to the parameters from Exhibit 3, we need two more parameters to compute the subjective values for each of the three options. These additional parameters are the fraction of wealth represented by the executive option (λ) and the relative risk aversion parameter (γ). Both parameters are also sampled uniformly $\lambda \in [0.05, 0.95]$ and $\gamma \in [0.50, 5]$ in order to cover a wide range of parameters.

Subjective values	Mean	RMSE	Avg Score
Power option P_T	0.4520	0.3503	2.59
Standard call C_T	0.4155	0.3332	1.73
Asian option A_T	0.4406	0.3364	1.69

EXHIBIT 6:
Summary statistics of the subjective values for our sample of 10,000 options. Payoffs are sorted in decreasing order of the average score.

Based on 10,000 sets of parameters, we compute the subjective values for each option payoff. We then assign a score to each payoff based on the rank of its subjective value in increasing order. The smallest subjective value is assigned a score of 1 and the largest one is assigned a score of 3. Results are reported in Exhibit 6. Note that in each simulation,

the cost spent by the company is the same so that the comparison among the three options is fair.

Note that if we were to compare the power option and the Asian option only, then the power option would always have a higher subjective value in all the simulations. In Exhibit 6, the standard call can outperform the power option for some of the simulations explaining a score of 2.59 instead of 3 for the power option. For convenience, we reproduce Table 2 of Tian [2013] in Appendix C (see Exhibit 11). In this table, we have added the corresponding results for the power options. The power option has a consistently higher subjective value than the Asian option.

4.2   Incentive Effectiveness

In this section, we compare the incentive properties of the three executive options (the Asian option, the power option and the standard call option). To standardize the comparison we assume that the initial company's cost is the same for the three options. Similar to Tian [2013], our measures of performance correspond to the pay-performance sensitivity (PPS) measure used by Jensen and Murphy [1990]. The total certainty equivalent value (TCE) of the executive's portfolio invested in the risk-free asset and the options is

$$\text{TCE} = (1 - \lambda)W_0 + n\text{CE} \tag{16}$$

Recall that

$$u\left(1 - \lambda + \lambda\frac{\text{CE}}{c(V_T)}\right) = E_{\mathbb{P}}\left[u\left(1 - \lambda + \lambda\frac{V_T}{c(V_T)}e^{-rT}\right)\right] \tag{17}$$

where V_T refers to any of the terminal payoffs considered (A_T, P_T or C_T), and $c(V_T)$ is its corresponding risk-neutral price (respectively $c(A_T)$, $c(P_T)$ and $c(C_T)$). The various incentive measures all arise from a combination of (16) and taking partial derivatives of both sides of (17) with respect to the parameter of interest.

The incentives to increase stock price, volatility, and dividends are measured by their respective sensitivities defined as the ratio of the percentage change in the executive's

total wealth to the percentage change in the stock price (PPS), volatility (VS), and dividend (DS). Their expressions are given in Tian [2013] and reproduced in Appendix A for completeness.

We first consider the incentive to increase the stock price. We use the rescaled pay-performance sensitivity (PPS) measure defined in (18) (Appendix A) as proposed by Tian [2013]. Based on the same sample of 10,000 sets of parameters, we compute the pay-performance sensitivity for each of the three options. Then, for each of the 10,000 sets of pay-performance sensitivities, we compute the score of each payoff as the rank of its pay-performance sensitivity in increasing order, the smallest one being assigned a score of 1 and the largest one being assigned a score of 3. Results are given in Exhibit 7. The vanilla call option produces the least incentive to increase share price. Separately, the Asian executive option and the power option already deliver high incentives to increase share price (with an average score of 2.17 and 2.83 respectively).

Pay-performance sensitivity (PPS)	Mean	RMSE	Avg Score
Power option P_T	0.6744	0.6055	2.83
Asian option A_T	0.6657	0.5973	2.17
Standard call C_T	0.3934	0.3628	1.00

EXHIBIT 7:
Summary statistics of the pay performance sensitivities values calculated for our sample of 10,000 option costs. Payoffs are sorted in decreasing order of the average score.

For convenience, we reproduce Table 3 in Tian [2013] in Appendix C (see Exhibit 12) and compare his results with the power option. While there is no clear dominance for the Asian option and the standard call, the power option outperforms the Asian executive option on the basis of pay performance sensitivities across all sets of parameters investigated by Tian [2013]. Note that even though Exhibit 7 is presented in a compact form it covers a wide range of possible parameter values. It is clear from this analysis that the power option is better than the Asian option.

We next consider the incentive to increase volatility. A positive value of the volatility

sensitivity means that the particular option incentivizes the executive to increase share volatility, a presumably undesirable effect. Hence, the smaller the value of volatility sensitivity, the better it is. We compute the score of each payoff as the rank of its volatility sensitivity in decreasing order, the largest one being assigned a score of 1 and the smallest one being assigned a score of 3. This way, the more desirable property of having a negative volatility sensitivity is assigned a higher score. From Exhibit 8, the power option clearly dominates the Asian executive option and the standard call option. In fact, the volatility sensitivity for the power option is much smaller than all of the other options across all parameters.

Volatility Sensitivity (VS)	Mean	RMSE	Avg Score
Power option P_T	-2.9675	3.5158	2.84
Standard call C_T	-0.1990	0.2146	1.68
Asian option A_T	-0.1904	0.2631	1.49

EXHIBIT 8:
Summary statistics of the volatility sensitivities values calculated for our sample of 10,000 options. Payoffs are sorted in decreasing order of the average score.

As a last comparison criteria, Tian [2013] also examines the incentive to reduce dividend payments. Johnson and Tian [2000a] point out that since most executive options are not protected against dividends that would reduce terminal stock price, there is an incentive for executives to reduce dividend payments in order to increase their dividend option payoffs. As a practical matter, this may not be an important property because it is well-known that corporations show extreme reluctance to reduce dividends, even under adverse financial conditions.

We examine the average scores of the dividend sensitivity calculated using the same set of parameters sampled from Exhibit 3. For each of the 10,000 samples, we compute the score of each payoff as the rank of its dividend sensitivity in increasing order, the smallest one being assigned a score of 1 and the largest one being assigned a score of 3. Exhibit 9 gives the results. It shows that along this dimension the power option provides

the strongest incentive to reduce dividends: the average score of the power option is almost identically 1, the lowest possible score. Out of the sample of 10,000 sets of parameters, the power option received a score of 1 (lowest value of dividend sensitivity) in 9,968 of them. As explained above, we do not regard this as a critical disadvantage of the power option.

DS	Mean	RMSE	Avg Score
Asian option A_T	-0.0207	0.0248	2.89
Standard call C_T	-0.0243	0.0297	2.11
Power option P_T	-0.2954	0.3508	1.00

EXHIBIT 9:
Summary statistics of the dividend sensitivities values calculated for our sample of 10,000 options. Payoffs are sorted in decreasing order of the average score.

For convenience, Exhibit 10 below reproduces the scores calculated, as well as the aggregate score for all criteria which is just the straight average. If we were to omit the sensitivity to dividend payments, the results would be even more striking.

Payoff	Cost	Subjective Value	Pay Performance Sensitivity	Volatility Sensitivity	Dividend Sensitivity	Aggregate Score
Power option P_T	3.00	2.59	2.83	2.84	1.00	2.45
Asian option A_T	2.00	1.69	2.17	1.49	2.89	2.05
Standard call C_T	1.00	1.73	1.00	1.68	2.11	1.50

EXHIBIT 10:
Average score of each option across different criteria, sorted in decreasing order by the aggregate score.

5   Conclusions

This paper proposed a new form of executive stock option that has appealing properties. This new contract is a power option. It has advantages over Tian's [2013] Asian option

and over a traditional call option. The new power option is cheaper than the Asian option to the company. For a given issuance cost to the company, the power option has superior subjective value and incentive properties compared to the Asian option. The comparison is based on the firm's cost of issuance, subjective values to executives, and incentives to increase share price, volatility, and dividends. This comparison is comprehensive and uses a wide range of sampled parameters.

The power option is actually not very complicated. Given that it is cheaper to the company and that for a given cost at issuance, it has also a larger subjective value to executives, it is clear that this efficiency gain can be used to improve the welfare of both parties, the company and the executive.

It is also appropriate to mention some extensions and limitations. We have used the geometric average Asian option as our benchmark because in this case both the Asian option and its power option counterpart have simple closed from solutions. The same analysis goes though if we were to use the arithmetic Asian option as a benchmark but in this case neither the Asian option nor its cost efficient counterpart have closed form solutions.[7] We use the Black-Scholes market model but results can be extended to more realistic models using numerical techniques.

Asian options are quite common in financial markets and well understood by market participants. For example many defined benefit pension plans base the pension on the average salary of the last few years. The new power options may not be as easy to understand and may be viewed as more abstract and complicated. Although our proposed power options have the same ex-ante distribution as the benchmark Asian option, there will be scenarios when one of them is the money and the other is out of the money. The Legg Mason example in Section 3 provides an example. So, these two options are not equivalent on an ex-post post basis and this might be a source of concern to some executives when the Asian option outperforms the power option if they hold the power options.

The power option requires an estimate of the stock's expected return. The expected

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[7] However, results would be approximately the same as the distribution of the arithmetic average is close to that of the geometric average.

return is difficult to estimate so pricing these new options is more challenging than pricing say a standard call or an Asian option. Despite these limitations, we think that the proposed power option merits consideration as an alternative to standard call options and Asian options in the structure of executive compensation.

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A   Expressions of Sensitivities

The expression of the pay-performance sensitivity (PPS) is also given in (17) of Tian [2013], and the volatility sensitivity appears in (20) of Tian [2013].

$$\text{PPS} = \frac{\partial(\text{TCE})}{\partial S_0}\frac{S_0}{\text{TCE}} = \frac{E_{\mathbb{P}}\left[u'\left[(1-\lambda)+\lambda\frac{V_T}{c(V_T)}e^{-rT}\right]\frac{\partial V_T}{\partial S_0}\right]}{u'\left[(1-\lambda)+\lambda\frac{\text{CE}}{c(V_T)}\right]}\frac{\lambda S_0 e^{-rT}}{(1-\lambda)c(V_T)+\lambda\text{CE}} \quad (18)$$

$$\text{VS} = \frac{\partial(\text{TCE})}{\partial \sigma}\frac{\sigma}{\text{TCE}} = \frac{E_{\mathbb{P}}\left[u'\left[(1-\lambda)+\lambda\frac{V_T}{c(V_T)}e^{-rT}\right]\frac{\partial V_T}{\partial \sigma}\right]}{u'\left[(1-\lambda)+\lambda\frac{\text{CE}}{c(V_T)}\right]}\frac{\lambda \sigma e^{-rT}}{(1-\lambda)c(V_T)+\lambda\text{CE}} \quad (19)$$

$$\text{DS} = \frac{\partial(\text{TCE})}{\partial q}\frac{q}{\text{TCE}} = \frac{E_{\mathbb{P}}\left[u'\left[(1-\lambda)+\lambda\frac{V_T}{c(V_T)}e^{-rT}\right]\frac{\partial V_T}{\partial q}\right]}{u'\left[(1-\lambda)+\lambda\frac{\text{CE}}{c(V_T)}\right]}\frac{\lambda q e^{-rT}}{(1-\lambda)c(V_T)+\lambda\text{CE}} \quad (20)$$

B   Proof of Proposition 1

This proposition consists of two parts. The first part is based on the results in Bernard et al. [2014], they show that in the Black-Scholes setting, when $\mu \geqslant r$, for any cumulative distribution function (cdf) F, the unique[8] cost-efficient payoff with cdf F is given by

$$V_T^{\star} = F^{-1}\left(F_S\left(S_T\right)\right) \quad (21)$$

where the pseudo inverse of F is defined as $F^{-1}(y) = \min\{x \mid F(x) \geq y\}$. In other words, let V_T be any other executive compensation with the same cdf F, $c\left(V_T^{\star}\right) \leq c(V_T)$. The expression of P_T follows applying formula (21) to the Asian executive compensation A_T that is defined in (3). We omit the derivations as it is a straightforward extension to include a dividend yield in the example of the cost-efficient alternative to a Geometric Asian option derived by Bernard et al. [2014].

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[8]Uniqueness is only almost surely: it means that any payoff equal to $V_T^{\star}$ everywhere except on a set with probability measure zero would also be cost-efficient.

The second part of Proposition 1 requires a proof. For the cost-efficient alternative P_T (with certainty equivalent CE_{Power} and cost $c(P_T)$) compared to the proposed Asian compensation A_T (with certainty equivalent CE_{Asian} and cost $c(A_T)$), observe that

$$u\left(1 - \lambda + \lambda\frac{\text{CE}_{Power}}{c(P_T)}\right) = E_{\mathbb{P}}\left[u\left(1 - \lambda + \lambda\frac{P_T}{c(P_T)}e^{-rT}\right)\right] \qquad (22)$$

$$u\left(1 - \lambda + \lambda\frac{\text{CE}_{Asian}}{c(A_T)}\right) = E_{\mathbb{P}}\left[u\left(1 - \lambda + \lambda\frac{A_T}{c(A_T)}e^{-rT}\right)\right] \qquad (23)$$

From the increasing property of the utility function and the fact that $c(P_T) < c(A_T)$,

$$E_{\mathbb{P}}\left[u\left(1 - \lambda + \lambda\frac{P_T}{c(P_T)}e^{-rT}\right)\right] > E_{\mathbb{P}}\left[u\left(1 - \lambda + \lambda\frac{P_T}{c(A_T)}e^{-rT}\right)\right]$$

$$= E_{\mathbb{P}}\left[u\left(1 - \lambda + \lambda\frac{A_T}{c(A_T)}e^{-rT}\right)\right]$$

The last equality comes from the fact that P_T and A_T have the same probability distribution. Using (22) and (23), we have that

$$u\left(1 - \lambda + \lambda\frac{\text{CE}_{Power}}{c(P_T)}\right) > u\left(1 - \lambda + \lambda\frac{\text{CE}_{Asian}}{c(A_T)}\right)$$

Finally, as u is increasing, the result is proved. $\qquad\qquad\square$

C Appendix: Numerical Exhibits

This Appendix reproduces several of the Tables in Tian's [2013] paper with the addition of the power option.

Risk aversion γ	Weight in options λ	Subjective value as a fraction of risk-neutral value (%)		
		Call C_T	Power P_T	Asian A_T
Panel A: At-the-money options ($K = S_0$)				
2	0.90	11.1	11.2	11.0
2	0.75	25.8	26.7	26.0
2	0.50	51.5	54.4	52.4
2	0.25	87.3	92.2	87.5
3	0.90	5.3	5.3	5.2
3	0.75	14.2	14.4	14.2
3	0.50	33.6	35.0	34.2
3	0.25	66.3	70.3	67.5
Panel B: In-the-money options ($K = 0.8S_0$)				
2	0.90	14.7	18.6	18.2
2	0.75	32.0	39.4	38.2
2	0.50	59.5	69.9	67.1
2	0.25	94.3	104.6	99.1
3	0.90	7.0	8.6	8.5
3	0.75	18.1	22.2	21.9
3	0.50	40.3	48.6	47.2
3	0.25	74.2	85.3	81.6
Panel C: Out-the-money options ($K = 1.2S_0$)				
2	0.90	8.6	7.1	7.0
2	0.75	21.1	18.2	17.9
2	0.50	44.8	41.1	39.8
2	0.25	80.4	78.2	74.4
3	0.90	4.2	3.4	3.4
3	0.75	11.4	9.6	9.5
3	0.50	28.3	25.0	24.6
3	0.25	59.2	55.9	53.9

EXHIBIT 11:
Table 2 of Tian [2013]. Parameters used: $T = 10$, $K \in [80, 120]$, $r = 4\%$, $q = 2\%$, $\mu = 8\%$, $\sigma = 30\%$. **The relative risk aversion is 2 or 3 and the weight in options varies from 25% to 90%.**

Weight in options λ	Strike price K	Pay-to-Performance Sensitivity (PPS)		
		Call C_T	Power P_T	Asian A_T
Panel A: Moderate level of risk aversion ($\gamma = 2$)				
0.25	50	0.307	0.447	0.431
0.25	75	0.323	0.539	0.518
0.25	100	0.326	0.574	0.553
0.25	125	0.320	0.551	0.532
0.50	50	0.527	0.833	0.811
0.50	75	0.529	0.940	0.915
0.50	100	0.508	0.903	0.883
0.50	125	0.476	0.782	0.769
0.75	50	0.792	1.346	1.321
0.75	75	0.749	1.394	1.372
0.75	100	0.681	1.205	1.192
0.75	125	0.610	0.960	0.954
0.90	50	1.059	1.944	1.926
0.90	75	0.937	1.792	1.784
0.90	100	0.811	1.410	1.411
0.90	125	0.701	1.062	1.067
Panel B: High level of risk aversion ($\gamma = 3$)				
0.25	50	0.259	0.410	0.397
0.25	75	0.261	0.465	0.449
0.25	100	0.253	0.453	0.440
0.25	125	0.238	0.398	0.388
0.50	50	0.417	0.743	0.725
0.50	75	0.390	0.740	0.725
0.50	100	0.353	0.629	0.620
0.50	125	0.315	0.497	0.493
0.75	50	0.573	1.140	1.124
0.75	75	0.493	0.962	0.957
0.75	100	0.421	0.736	0.734
0.75	125	0.361	0.547	0.547
0.90	50	0.680	1.462	1.460
0.90	75	0.551	1.080	1.085
0.90	100	0.455	0.784	0.786
0.90	125	0.383	0.566	0.570

EXHIBIT 12:

Table 3 of Tian [2013]. Parameters used: $T = 10$, $K \in [50, 125]$, $r = 4\%$, $q = 2\%$, $\mu = 8\%$, $\sigma = 30\%$. The relative risk aversion is 2 or 3 and the weight in options varies from 25% to 90%.